Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1.   Determine if **w** is in the subspace spanned by **v, u**

$$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix}.$$

2.   Determine if **w** is in the C(A) if $\mathbf{A} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$, where

$$v_1 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}.$$

3.   Determine if **w** is in Nul(A) if $\mathbf{A} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$, where

$$v_1 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}.$$

4.   Find a Basis for the Null space and a Basis for the Column space of A.

$$\begin{bmatrix} 1 & 1 & 2 & 5 \\ 1 & -1 & 1 & 0 \\ 2 & 1 & 3 & 7 \end{bmatrix}$$

5.   Find the Rank and the Nullity of Matrix A in problem 4.

6.   Construct a matrix that has a rank equal 3 and Nullity 2, then give the Basis for the Column space and a Basis for the Null space of your matrix.