Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1.    For each subspace find a basis and state the dimension.

a.    $\left\{ \begin{bmatrix} s-3t \\ s+t \\ 2t \end{bmatrix}, s \text{ and } t \text{ in R} \right\}$

b.    $\left\{ \begin{bmatrix} s \\ s+t \\ 2t \end{bmatrix}, s \text{ and } t \text{ in R} \right\}$

c.    $\{(a,b,c) : a - 3b + c = 0, \ b - 2c = 0, \ 2b - c = 0\}$

2.    Find the dimension of the of the subspace H spanned by the given set of vectors.

$$b_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \ b_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \ b_3 \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

4.      Determine the dimension of the Null Space and the Column space

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$$

5.      Find the dimension of the subspace of aal vectors in $R^3$ whose first and third entries are equal.

6.      Consider the first four Laguerre polynomials in the set $\{1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3\}$

a.      Prove that the set S forms a linearly independent set hence it forms a basis for the set of polynomials $P_2$.

b.      Find the coordinate vector of $p(t) = -6 - 12t + t^2$ relative to B.