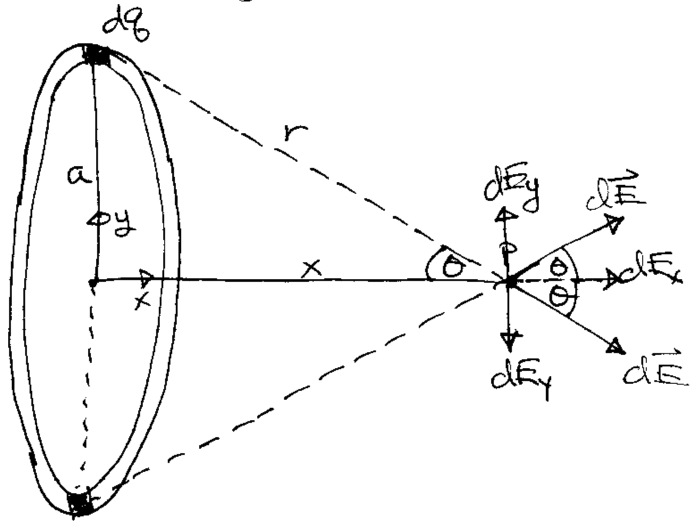
Ex. $\vec{E}$-field Due to a ring of charge $+Q$ distributed uniformly.



$r = (a^2+x^2)^{1/2}$

By symmetry $E_y = 0$

$dE_x = dE \cos\theta$

$dE_x = \frac{K\,dq}{r^2}\left(\frac{x}{r}\right) = \frac{K\,dq\,x}{r^3}$

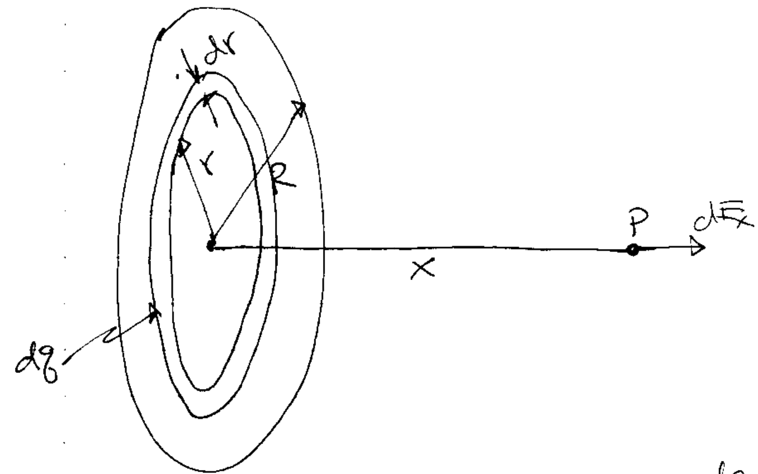
$dE_x = \frac{Kx(dq)}{(a^2+x^2)^{3/2}}$

$E_x = \frac{Kx}{(a^2+x^2)^{3/2}} \int dq$

$E_x = \frac{KQx}{(a^2+x^2)^{3/2}}$

If $x \gg a$, then $E_x \approx \frac{KQ}{x^2}$ ✓

Ex. $\vec{E}$-field Due to a disk of charge $+Q$ distributed uniformly.



$$\frac{Q}{\pi R^2} = \frac{dq}{dA}$$

$$dq = \left(\frac{Q}{\pi R^2}\right) dA$$

$$dq = \sigma\, dA$$

σ = surface charge density

$$dq = \sigma(2\pi r\,dr)$$

$$dE_x = \frac{Kx(dq)}{(r^2+x^2)^{3/2}} \quad (\vec{E}\text{-field due to ring})$$

$$dE_x = Kx\sigma\pi \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$

let $u = r^2+x^2$
$du = 2r\,dr$
$u_i = x^2$
$u_f = R^2+x^2$

$$E_x = Kx\sigma\pi \int_0^R \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$

$$= Kx\sigma\pi \int_{x^2}^{R^2+x^2} \frac{du}{u^{3/2}}$$

$$= Kx\sigma\pi \left. \frac{u^{-1/2}}{-1/2} \right|_{x^2}^{R^2+x^2}$$

$$= -2Kx\sigma\pi \left[\frac{1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right]$$

$$\vec{E} = 2K\sigma\pi \left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]\hat{i}$$

$$\lim_{R\to\infty} \vec{E} = 2K\sigma\pi \lim_{R\to\infty} \Big[\underbrace{1 - \frac{x}{\sqrt{R^2+x^2}}}_{1} \Big] \hat{i}$$

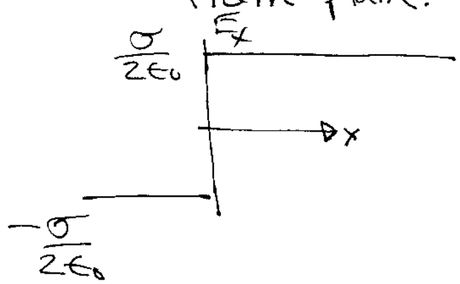
$$\vec{E} \approx 2K\sigma\pi \hat{i}$$

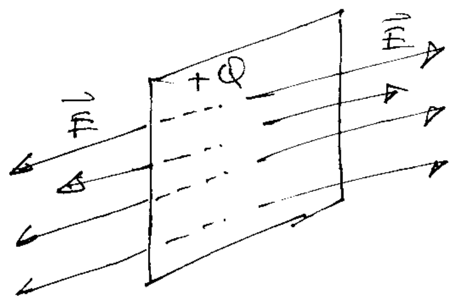
$$\vec{E} \approx 2 \frac{1}{4\cancel{\pi}\epsilon_0} \sigma \cancel{\pi} \hat{i}$$

$$\boxed{\vec{E} \approx \frac{\sigma}{2\epsilon_0} \hat{i}}$$

$\vec{E}$-field Due To an Infinite plane of charge

- field is uniform
- field is $\perp$ to plane
- field is independent of distance from plane.





In the limit as $x \gg R$, $\vec{E}$ should approach that of a point charge:

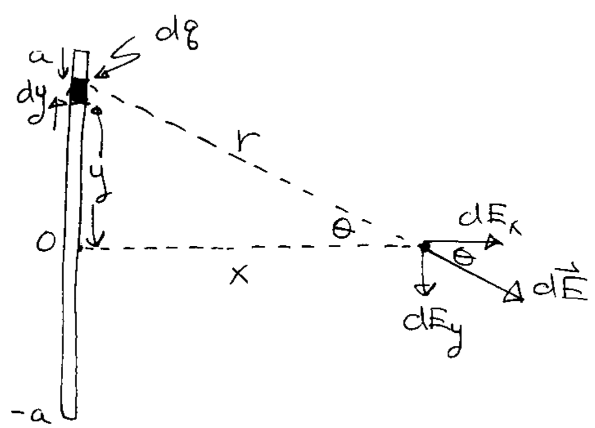
$$\vec{E} = 2K\sigma\pi \Big[1 - \frac{\cancel{x}}{\cancel{x}\sqrt{\frac{R^2}{x^2}+1}} \Big] \hat{i} \qquad \left(\frac{R^2}{x^2}+1\right)^{-1/2} \approx 1 - \frac{1}{2}\frac{R^2}{x^2} + \dots$$

$$\vec{E} \approx 2K\frac{Q}{\cancel{\pi}R^2}\cancel{\pi} \Big[\cancel{1} - \Big(\cancel{1} - \frac{1}{2}\frac{R^2}{x^2}\Big) \Big] \hat{i}$$

$$\vec{E} \approx \frac{\cancel{2}KQ}{\cancel{R^2}} \Big(\frac{1}{\cancel{2}} \frac{\cancel{R^2}}{x^2} \Big) \hat{i}$$

$$\vec{E} \approx \frac{KQ}{x^2} \hat{i} \checkmark$$

B. $\vec{E}$-field off the axis of line of charge on $\perp$ bisector.



$r = \sqrt{x^2+y^2}$

Let's divide the line charge into infinitesimal segments each of which acts like a point charge. Let the length of a typical segment to be dy at a height y

Since charge is distributed uniformly:

$$\frac{Q}{2a} = \frac{dq}{dy} \Rightarrow dq = \left(\frac{Q}{2a}\right)dy$$

$$\lambda = \frac{Q}{2a} \text{ (linear charge density)}$$

$$dq = \lambda dy$$

$$dE = \frac{K\,dq}{r^2} = \frac{K\lambda\,dy}{x^2+y^2}$$

If one considers the E-field at the same point due to dq located at -y, then the y-components of E- cancel. Thus, by symmetry $E_y = 0$.

$dE_x = dE\cos\theta$

$$dE_x = \frac{K\lambda dy}{x^2+y^2}\,\frac{x}{(x^2+y^2)^{1/2}}$$

$$dE_x = K\lambda x \frac{dy}{(x^2+y^2)^{3/2}} \qquad \int \frac{dy}{(y^2+x^2)^{3/2}} = \frac{y}{x^2(y^2+x^2)^{1/2}}$$

$$\int dE_x = 2K\lambda x \int_0^a \frac{dy}{(x^2+y^2)^{3/2}}$$

$$E_x = 2K\lambda \cancel{x} \left.\frac{y}{x^{\cancel{2}}(y^2+x^2)^{1/2}}\right|_0^a$$

$$E_x = 2K\lambda \left[\frac{a}{x(a^2+x^2)^{1/2}}\right]$$

$$E_x = \frac{\cancel{2}KQ}{\cancel{2a}}\,\frac{\cancel{a}}{x\sqrt{a^2+x^2}}$$

$$\boxed{E = \frac{KQ}{x\sqrt{a^2+x^2}}\,\hat{i}}$$

for $x \gg a$, $E_x \approx \frac{KQ}{x(x)} = \frac{KQ}{x^2}$ ✓

$$\vec{E} = \frac{KQ}{x\sqrt{a^2+x^2}}\,\hat{i} \qquad \text{but } \lambda = \frac{Q}{2a} \Rightarrow Q = \lambda 2a$$

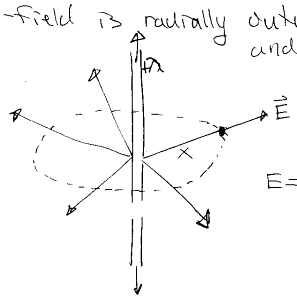
$$\vec{E} = \frac{K2\lambda a}{x\sqrt{a^2+x^2}}\,\hat{i}$$

$$\vec{E} = \frac{2K\lambda \cancel{a}}{\cancel{a}x\sqrt{1+\frac{x^2}{a^2}}}\,\hat{i}$$

$$\lim_{a\to\infty} \vec{E} = \frac{2K\lambda}{x} \lim_{a\to\infty} \frac{1}{\sqrt{1+\frac{x^2}{a^2}}} = \frac{2K\lambda}{x}(1)$$

$$\boxed{\vec{E} \approx \frac{2K\lambda}{x}\,\hat{i}}$$ E-field due to an infinite line of charge

E-field is radially outward if λ is positive and radially inward if λ is negative.



$E = \frac{2K\lambda}{x}$

$$\tan\theta = \frac{y}{x} \Rightarrow y = x\tan\theta$$

$$dy = x\sec^2\theta\, d\theta$$

$$= \int \frac{x\sec^2\theta\, d\theta}{(x^2\tan^2\theta + x^2)^{3/2}}$$

$$= \int \frac{x\sec^2\theta\, d\theta}{x^3(\tan^2\theta+1)^{3/2}}$$

$$= \frac{1}{x^2}\int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^{3/2}}$$

$$= \frac{1}{x^2}\int \frac{d\theta}{\sec\theta} = \frac{1}{x^2}\int \cos\theta\, d\theta$$

$$= \frac{1}{x^2}\sin\theta$$

$$= \frac{1}{x^2}\,\frac{y}{\sqrt{x^2+y^2}}$$ ✓