# Adding Vectors Graphically and Component Method

## Objective
To learn how to add vectors graphically and component method and compare with expected resultant vector.

## Equipment
1. protractor
2. ruler
3. paper
4. force table
5. set of masses
6. triple-beam balance

## Theory

**DEF:** A <u>vector</u> is a quantity that has both magnitude and direction.

**DEF:** A <u>scalar</u> is a quantity that has magnitude but NO direction.

| Ex. | Vectors | Ex. | Scalars |
|-----|---------|-----|---------|
| | Force | | Temperature |
| | Velocity | | Time |
| | Displacement | | Mass |
| | Momentum | | Speed |

### *Vector Notation*

**A** – Boldface letters

$\vec{A}$ - Arrow above letter

A – Magnitude of vector **A**

A vector is defined graphically by an arrow whose length is proportional to the magnitude of the vector quantity. The direction of the arrow points in the direction of the vector quantity.

### *Adding Vectors Graphically*

Consider adding two vectors A and B graphically. The two vectors are shown below.
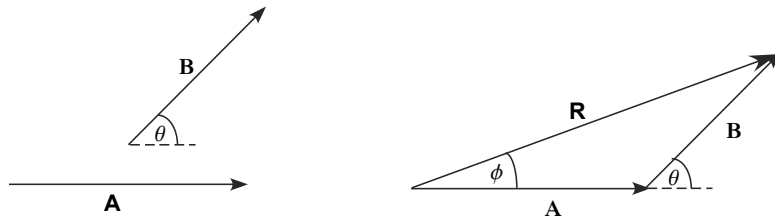


1. Select an appropriate scale. (Ex. 20 cm = 5 N)
2. Draw vector **A** to scale and in the proper direction.

3. Draw vector **B** to the same scale with its tail at the tip of **A** and in the proper direction.
4. The resultant vector **R** = **A** + **B** is the vector drawn from the tail of vector **A** to the tip of vector **B**.
5. Calculate the magnitude of the resultant vector **R** using the selected scale and measure its direction with a protractor.
6. This same process applies if you add more than two vectors.

This method of adding vectors graphically is also referred to as the head-to-tail method, analytical method, and geometric method.

**Ex**. A jogger runs 2.0 km due east, then 1.0 km at $45^\circ$ north of east, and finally 0.5 km due north. Calculate the displacement of the jogger.

     *Scale: 50cm = 2 km*

     ANS: R ≈ 2.98 km, θ ≈ $24^\circ$

## Adding Vectors Using Component Method

Consider adding three 2-D vectors **A**, **B**, and **C**:

     **A** = $A_x$**i** + $A_y$**j**
     **B** = $B_x$**i** + $B_y$**j**
     **C** = $C_x$**i** + $C_y$**j**

1. Add the x-components and y-components of each vector to obtain the resultant vector **R** in unit vector notation.

     **R** = **A** + **B** + **C** = ($A_x$**i** + $A_y$**j**) + ($B_x$**i** + $B_y$**j**) + ($C_x$**i** +$C_y$**j**)

**R** = ($A_x$ + $B_x$ + $C_x$) **i** + ($A_y$ + $B_y$ + $C_y$) **j**
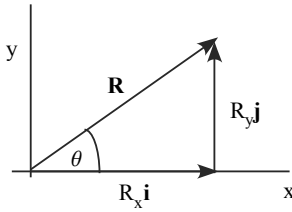
$$R_x = A_x + B_x + C_x$$

$$R_y = A_y + B_y + C_y$$

$$\mathbf{R} = R_x\,\mathbf{i} + R_y\,\mathbf{j}$$

2. Calculate the magnitude of the resultant vector **R**.

$$R = \sqrt{R_x^2 + R_y^2}$$

3. Calculate the direction of **R** relative to the positive x – axis.



$$\tan \theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

4. Same procedure applies if you add more than 3 vectors. However, if the vectors are 3D, then you must specify the direction of the resultant vector **R** relative to the positive x, y, and z axis.

## Procedure

### *Exercise 1*

1. A car travels 20 mi at $60^0$ north of west, then 35 mi at $45^o$ north of east.
2. Express each displacement vector in unit vector notation. Take the +x-axis due east and the +y-axis due north.
3. Use the component method to obtain the resultant displacement vector in unit vector notation. Calculate the magnitude and direction.
4. Add the displacements vectors graphically using an appropriate scale and coordinate system. Obtain the resultant vector and calculate the magnitude and direction.
5. Calculate the % error between the graphical and component method. Take the component method to be the expected value.

### *Exercise 2*

*Finding resultant force experimentally.*

1. Suppose a particle is acted on by the following three forces:

   $F_1 = m_1g$ @ $30^o$        ($m_1 = 300$ gr)
   $F_2 = m_2g$ @ $110^o$      ($m_2 = 450$ gr)
   $F_3 = m_1g$ @ $230^o$      ($m_3 = 400$ gr)

   (Our particle will be a metal ring centered on the force table)

2. Measure the three masses $m_1$, $m_2$, and $m_3$ on the triple-beam balance and calculate the forces $F_1$, $F_2$, and $F_3$ associated with each hanging mass.

3. Add the masses to the pulleys:

    a. 300 g @ $30°$
    b. 450 g @ $110°$
    c. 400 g @ $230°$

4. Find the resultant force $F_R'$ that will balance the three forces by adding a fourth mass to the force table at the appropriate angle.

5. Based on the resultant force $F_R'$ from step (4) above, determine the resultant force $F_R$ on the particle. $F_R$ is the expected value.

## Finding resultant force using Component Method

1. Express each force $F_1$, $F_2$, and $F_3$ in unit vector notation. Take the origin to be at the center of the force table (at pivot point) with the +x axis along $0°$ and +y-axis along $90°$.
2. Use the component method to obtain the resultant force vector $F_{comp}$ in unit vector notation. Calculate the magnitude and direction.

## Finding resultant force using Graphically

1. Add the vectors $F_1$, $F_2$ and $F_3$ graphically using an appropriate scale and coordinate system.
2. Obtain the resultant vector $F_{grap}$. Calculate the magnitude and direction.
3. Calculate the % error between the graphical method, component method, and the expected value.